

# IEEE 754 Decimal Floating-Point --- in Binary

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Mahesh Bhat (Intel)  
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# Agenda

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- **Why Decimal FP?**
- **Natural Advantages, Intel's View**
- **Tale of Two Formats**
- **Summary**

# Why not use binary FP?

- binary fractions *cannot* exactly represent all decimal fractions
- $1.2 \times 1.2 \rightarrow 1.44 ?$ 
  - 1.2 in a 32-bit binary float is actually:  
1.2000000476837158203125
  - and this squared is:  
1.440000057220458984375

# A financial example...

- 5% sales tax on a \$ 0.70 telephone call, rounded to the nearest cent
- $1.05 \times 0.70$  using binary double type is  
0.734999999999999998667732370449812151491641998291015625  
(should have been 0.735)
- rounds to \$ 0.73, instead of \$ 0.74

# Hence...

- Binary floating-point cannot be used for commercial applications
  - cannot match values computed by hand
  - cannot meet legal and financial requirements, which are based on 2,500+ years of decimal arithmetic
- So applications use decimal software floating-point packages...

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# Decimal FP Pros/Cons

- Advantage: Match Pencil & Paper Calculations
  - Decimal Fractions represented exactly
  - Wide precisions (16 and 34 digits) support exact calculation (no rounding) for modest formulas
- Disadvantage: Worst-Case Rounding error
  - Coarser rounding boundary, “Wobble”
- Disadvantage: Efficiency
  - Decimal FP logic is larger, consumes more power, and longer latency than Binary FP logic
  - Not quantified yet

# Intel's View of IEEE 754r Decimal FP

- Decimal Types are Important
  - Most monetary data in app code is naturally decimal
  - Standardized computation is important – same results
- Applications Experience with Software Decimal FP
  - Decimal cycle % is modest (<5%) in Monetary Apps
- IEEE 754r must enable efficient software library
  - Satisfies the broad market, for the foreseeable future
  - Needed to start adoption of the standard
- Formats must be software friendly
  - Efficient unpacking of components into binary
  - Exploit existing binary hardware
  - Efficient packing of components back to compact form

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# Decimal FP Encoding - DPD

Encoding numeric values

$$(-1)^s 10^{E-\text{bias}} c$$

For example  $8280.95 = 10^{-2} \times \underbrace{828}_{\text{integer}} \underbrace{095}_{\text{fraction}}$

Conceptually BCD

1000 0010 1000

0000 1001 0101

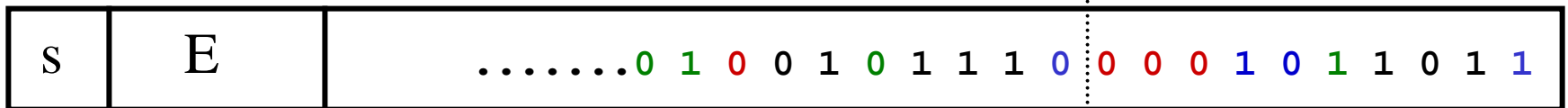
Packed into DPD

01 0 01 0 111 0

000 10 1 101 1

indicator

indicator



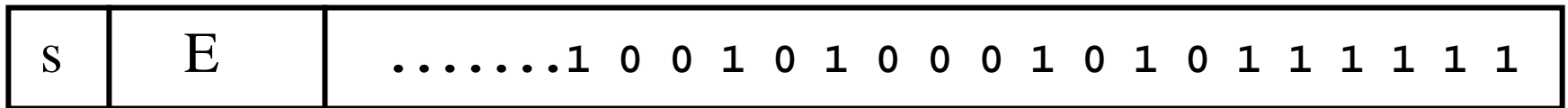
# Decimal FP Encoding - BID

Encoding numeric values

$$(-1)^s 10^{E-\text{bias}} c$$

For example  $8280.95 = 10^{-2} \times \underbrace{828095}$

11001010001010111111



# Software Performance of BID

1.5GHz Itanium2

BID

DPD

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TOTAL TIME (sec)

0.40

1.01

Decimal Time (sec)

0.15

0.65

latencies (avg. in cycles)

add

12

88

mul

12

95

rescale

40

101

Factors contributing to BID performance gap

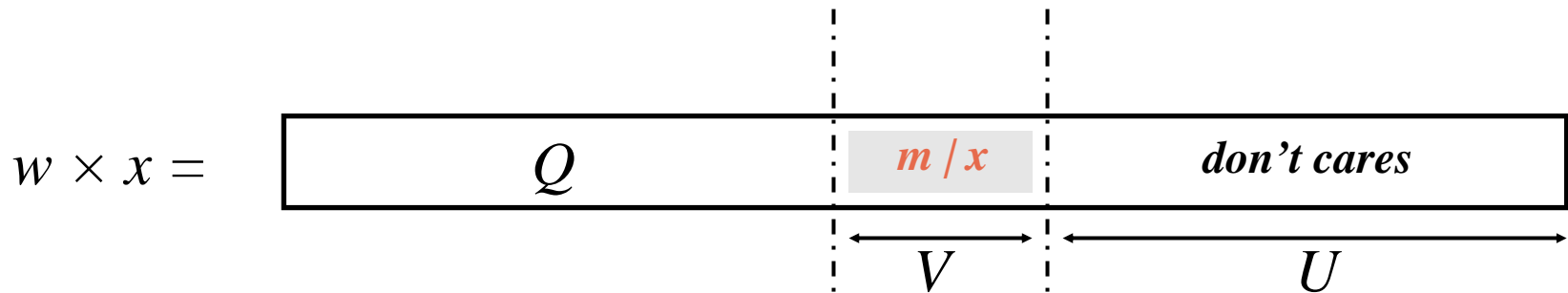
- Binary algorithms – use existing binary hardware
- Small pack/unpack overhead for BID <-> binary components

# Decimal Rounding BID

Let  $0 < x < 2^U$  be a dividend and  $0 < m < 2^V$  be a divisor.

Let  $x/m = Q + (r/m)$ , we wish to compute  $Q$  and  $r \equiv 0$

Let  $S = U + V$  and  $w = \text{ceil}(2^S / m) = 2^S / m + a/m$ ,  $0 < a < m$



# Evolution Key to Decimal

- BID is compatible with binary hardware
- Incremental hardware roadmap possible

**Increasing support**



Microcode + ROM

Existing multiplier, multiple pass (non-pipelined)

Existing multiplier with separate non-pipelined rounder

Existing multiplier with pipelined rounder

Separate unit

# Summary

- Decimal FP is important for Monetary Computations
  - Data is naturally decimal
  - Legally Required
  - Modest, but not negligible, profile in decimal operations
- Efficient Software Library
  - Satisfies the Broad Market, for the foreseeable future
  - Needed to foster adoption of the standard
- Binary-Integer Decimal (BID) Format Enables
  - An efficient software library
  - Efficient Rounding with proper algorithms
  - Hardware sharing with binary FP
  - Phased hardware implementation

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